CHAPTER 4

Moving Charges and Magnetism

MAGNETIC FIELD

A stationary charge does not experience any force near a current carrying wire. But a moving charge does. Two wires carrying current are attracted towards or repelled away from each other. Since a current carrying wire is electrically neutral, and electrostatic force acts on a charge whether or not it is in motion, these forces cannot be electrostatic in nature. Thus these events are conclusive proof of a force other than an electrostatic force, which we call magnetic force. Thus we say that a moving charge or a current produces a magnetic field around it in which only a moving charge will experience a magnetic force.

In order to describe any type of field, we must define its magnitude, or strength, and its direction.

Magnetic field is the region surrounding a moving charge in which its magnetic effects are perceptible on a moving charge (electric current). Magnetic field intensity is a vector quantity and also known as magnetic induction vector. The magnetic field $\vec{B}$ is also called **magnetic induction**, or **flux density**. It is represented by $\vec{B}$.

There are two methods of calculating magnetic field at some point. One is **Biot-Savart law**, which gives the magnetic field due to an infinitesimally small current carrying wire at some point and the another is **Ampere law**, which is useful in calculating the magnetic field of a symmetric configuration carrying a steady current.

The unit of magnetic field is weber/m² and is known as tesla (T) in the SI system.

BIOT SAVART LAW

Biot-Savart law gives the magnetic induction due to an infinitesimal current element.

❖ The magnetic field grows weaker as we move farther from its source. In particular, the magnitude of the magnetic field dB is inversely proportional to the square of the distance from the current element $Id\vec{L}$ (The direction of $d\vec{L}$ is same as the direction of current at that point).
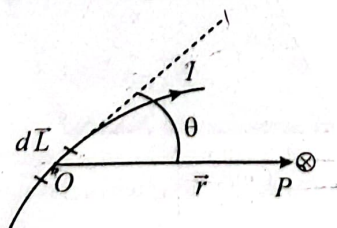
❖ The larger the electric current, the larger is the magnetic field. In particular, the magnitude of the magnetic field $d\vec{B}$ is proportional to the current I.

❖ The magnitude of the magnetic field $d\vec{B}$ is proportional to $\sin\theta$, where θ is the angle between the current element $Id\vec{L}$ and vector $\vec{r}$ (that points from the current element to the point in space where $d\vec{B}$ is evaluated). The direction of $d\vec{L}$ is the same as the direction of current at that point.

❖ The direction of the $\vec{B}$ is not radially towards or away from its source as the gravitational field and the electric field are from their sources. In fact, the direction of $d\vec{B}$ is perpendicular to both $Id\vec{L}$ and the vector $\vec{r}$.

These features of field $d\vec{B}$ can be written compactly as

$$d\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{Id\vec{L}\times\hat{r}}{r^2} = \left(\frac{\mu_0}{4\pi}\right)\frac{Id\vec{L}\times\vec{r}}{r^3}$$

Here, $(\mu_0/4\pi)$ is a constant of proportionality and $\hat{r}$ is unit vector in the direction of $\vec{r}$.

The constant μ_0 is called the permeability of free space or the permeability constant. Its value is

$$\mu_0 = 4\pi\times10^{-7}\ \text{Tm/A}$$

The SI unit of $\vec{B}$ is tesla (T) which is equivalent to Wb/m².

The dimensions of $\vec{B}$ are $[MT^{-2}A^{-1}]$

The magnitude of $d\vec{B}$ can be obtained by

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{I\,dL\sin\theta}{r^2}$$

The direction of $d\vec{B}$ is perpendicular to both $d\vec{L}$ and $\vec{r}$, i.e perpendicular to the plane of the paper. From the definition of the cross product, the field at the point P is directed into the paper. Such a field direction is shown by crossed circle ($\otimes$), according to convention.

The conventional sign for a field coming out of the plane, and perpendicular to it, is a *dot*, namely $\odot$. One way of remembering the sign convention is to imagine an arrow pointing in the direction of the vector. If the vector points out of the paper, we see the head of the arrow, namely, the $\odot$. If the vector points into the paper, we see the tail of the arrow, namely, $\otimes$.

The direction of $\vec{B}$ at a point in case of linear and circular current carrying conductors can be found as follows:
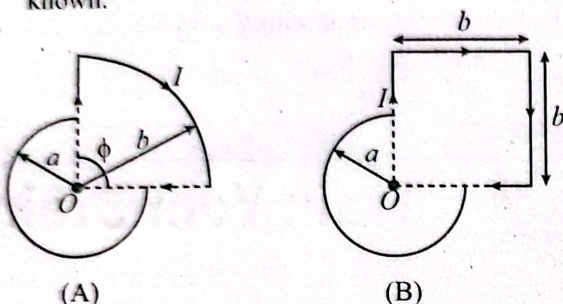
$$\vec{B}_{Net} = 3(\vec{B} \text{ due to one side})$$
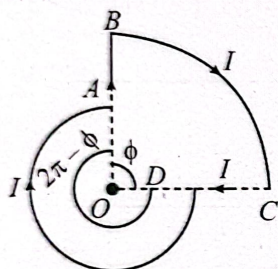
$$B_{net} = 3 \times \frac{\mu_0 i}{4\pi d}[\cos 60° + \cos 60°]$$

$$B_{net} = 3 \times 10^{-7} \times \frac{1}{\frac{a}{2}\tan 30°} \, 2\cos 30° = 4 \times 10^{-5} \text{ Wb/m}^2$$

Example 3: Find the magnetic induction of the field at the point O of a loop with current I, whose shape is illustrated in figure.

(i) In figure (A) below, the radii a and b, as well as the angle ϕ are known.

(ii) In figure (B) below, the radius a and the side b are known.

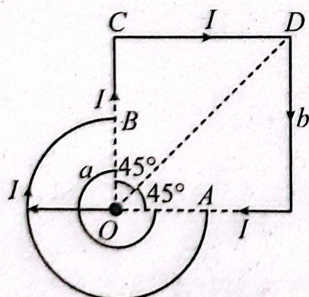(A)   (B)

Sol. (i)

B due to straight part $AB = 0 = B$ due to straight part CD

B due to curved part $AD = \left(\dfrac{2\pi - \phi}{2\pi}\right)\left(\dfrac{\mu_0 i}{2a}\right)$ (Into the plane of paper.)

B due to curved part $BC = \dfrac{\phi}{2\pi}\left(\dfrac{\mu_0 i}{2b}\right)$ (Into the plane of paper.)

$B_{net} = \dfrac{\mu_0 i}{4\pi}\left[\dfrac{2\pi - \phi}{a} + \dfrac{\phi}{b}\right]$ (Into the plane of paper.)

(ii)

B due to $BC = B$ due to $EA = 0$

B due to curved part $AB = \dfrac{\left(\dfrac{3\pi}{2}\right)}{2\pi}\dfrac{\mu_0 I}{2a}$

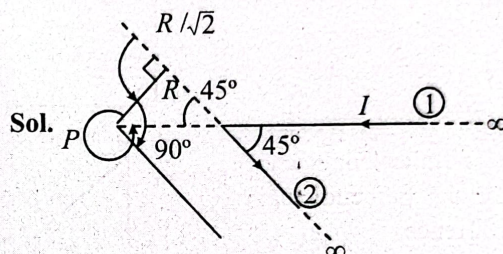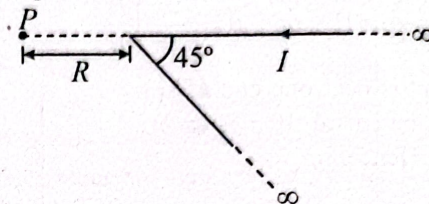$= \dfrac{3}{8}\dfrac{\mu_0 I}{a}$ Into the plane of paper

B due to $CD = \dfrac{\mu_0 i}{4\pi b}[\sin(0) + \sin(45°)]$

$= \dfrac{\mu_0 I}{4\sqrt{2}\pi b}$ Into the paper

B due to $DE = \dfrac{\mu_0 I}{4\sqrt{2}\pi b}$ Into the plane of paper.

$\vec{B}_{net} = \dfrac{\mu_0 I}{4\pi}\left[\dfrac{3\pi}{2a} + \dfrac{\sqrt{2}}{b}\right]$ Into the plane of paper

Example 4: A long straight wire, carrying current I, is bent at its midpoint to form an angle of $45°$. Find the magnetic induction at point P, distant R from point of bending.

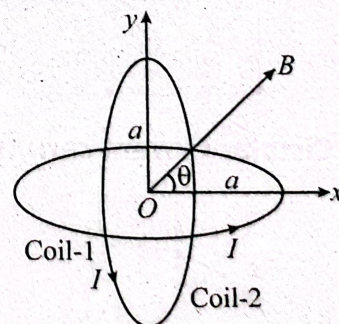
Sol.

$\vec{B}_P = \vec{B}_1 + \vec{B}_2$

$B_1 = 0$

For B_2: $\theta_1 = 2\pi - 45°$, $\theta_2 = 90°$

$B_2 = \dfrac{\mu_0 I}{4\pi \dfrac{R}{\sqrt{2}}}\times[\sin 90° + \sin(2\pi - 45°)]$

$= \dfrac{\mu_0 I}{4\pi R}\sqrt{2}\left[1 - \dfrac{1}{\sqrt{2}}\right]$

$B_2 = \dfrac{\mu_0 I}{4\pi R}[\sqrt{2} - 1]$; Thus, $B_P = \dfrac{\mu_0 I}{4\pi R}[\sqrt{2} - 1]$

Example 5: For the arrangement of figure the magnetic field at the center O will be

Sol. The two coils are perpendicular to each other. Coil 1 produce field along x axis and coil 2 produce field along y axis. Thus the resultant field will be

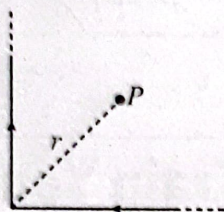$B = \sqrt{B_1^2 + B_2^2}$ making an angle θ, $\tan\theta = B_2/B_1$

For identical coils, $B = \sqrt{2}\left(\dfrac{\mu_0 NI}{2a}\right)$ and $\theta = 45°$
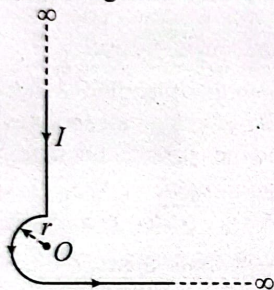
Concept Application

1. A long wire carrying current i is bent to form a right angle as shown in figure. Then magnetic induction at point P on the bisector of this angle at a distance r from the vertex is

   (a) $\dfrac{\mu_0 i}{2\pi R}$

   (b) $\dfrac{\mu_0 i}{4\pi r}(1+\sqrt{2})$

   (c) $\dfrac{\mu_0 i(1+\sqrt{2})}{2\pi r}$

   (d) $\dfrac{\mu_0 i}{4\pi R}$

2. In the figure, the magnetic induction at point O is

   (a) $\dfrac{\mu_0 i}{2\pi R}$

   (b) $\dfrac{\mu_0 i}{4\pi r}(1+\sqrt{2})$

   (c) $\dfrac{\mu_0 i(1+\sqrt{2})}{2\pi r}$

   (d) $\dfrac{\mu_0 i}{4\pi R}$

3. A wire of length L carrying current i is bent into circular loop with (i) one turn (ii) n turns. Find the ratio of magnetic induction at centre in above two cases.

   (a) $4 : n^2$

   (b) $3 : n^2$

   (c) $1 : n^2$

   (d) $2 : n^2$

4. A and B are the two concentric circular conductors of center O and carrying currents I_1 and I_2 as shown in the adjacent figure. If ratio of their radii is $1 : 2$ and ratio of the flux densities at O due to A and B is $1 : 3$ then the value of I_1/I_2 is
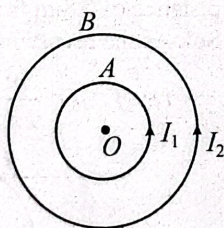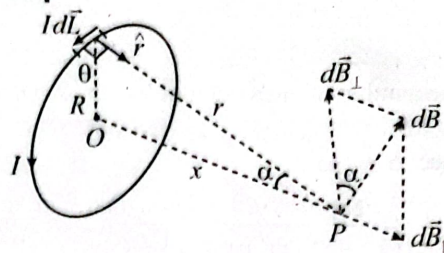

   (a) 1/6

   (b) 1/4

   (c) 1/3

   (d) 1/2

5. A current of one ampere is passed through a straight wire of length 2 metre. The magnetic field at a point in air at a distance of 3 m from one end of the wire but lying on the axis of the wire will be

   (a) $\mu_0/2\pi$

   (b) $\mu_0/4\pi$

   (c) $\mu_0/8\pi$

   (d) Zero

Field at an Axial Point of a Current carrying Circular Loop

Here, angle θ between the element $d\vec{L}$ and $\vec{r}$ is $\pi/2$ everywhere and r is same for all elements,

$$\therefore \quad \left| d\vec{L} \times \vec{r} \right| = r \, dL \sin\theta = r \, dL$$

$$\therefore \quad dB = \left(\frac{\mu_0}{4\pi}\right)\frac{I \, dL \, r}{r^3} = \left(\frac{\mu_0}{4\pi}\right)\frac{I \, dL}{r^2}$$

The field $d\vec{B}$ has components both along and perpendicular to the axis. However, if we consider the contributions of the current elements that are diametrically opposite, we see that their components normal to the axis $d\vec{B}_\perp$ will cancel.

But the component of $d\vec{B}$ along the axis is

$$dB_\| = dB \sin\alpha = \left(\frac{\mu_0}{4\pi}\right)\frac{I \, dL}{r^2}\left(\frac{R}{r}\right)$$

The total field is given by the integral of this expression over all elements,

$$\therefore \quad B = B_\| = \int dB_\| = \left(\frac{\mu_0}{4\pi}\right)\frac{IR}{r^3}\int_0^{2\pi R} dL = \left(\frac{\mu_0}{4\pi}\right)\frac{2I\pi R^2}{r^3}$$

However, as $r^2 = R^2 + x^2$, and if the coil has N turns, we have

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi R^2 IN}{(R^2 + x^2)^{3/2}}$$

Field at an Axial Point of a Solenoid

If many turns of an insulated wire are wound around a cylinder, the resulting coil is called a solenoid as shown in figure (a). The field at a point on the axis of a solenoid can be obtained by superposition of fields due to large number of identical coils all having their centre on the axis of the solenoid.

Solenoid
(a)

(b)
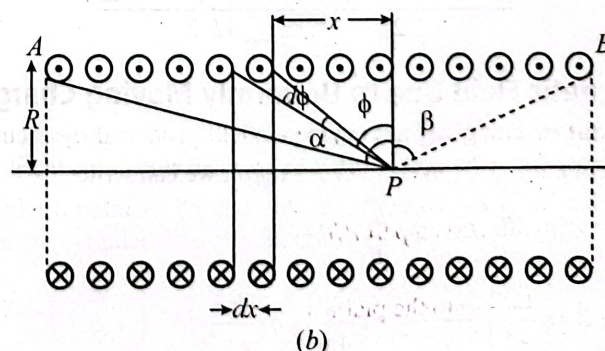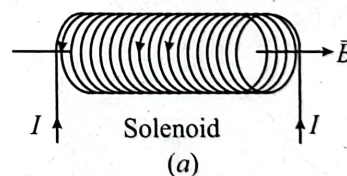
Considering a coil of width dx at a distance x from the point P on the axis, figure (b),

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi NIR^2}{(R^2 + x^2)^{3/2}}$$

If n is the number of turns per unit length, $N = n\,dx$ and $x = R\tan\phi$, i.e.,

$dx = R\sec^2\phi\,d\phi$, so

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi(n\,dx)IR^2}{(R^2 + R^2\tan^2\phi)^{3/2}} = \left(\frac{\mu_0}{4\pi}\right)(2\pi nI)\cos\phi\,d\phi$$

$$\therefore \quad B = \left(\frac{\mu_0}{4\pi}\right)(2\pi nI)\int_{-\alpha}^{\beta}\cos\phi\,d\phi$$

or $\quad B = \left(\frac{\mu_0}{4\pi}\right)(2\pi nI)[\sin\alpha + \sin\beta]$

$$\Rightarrow B = \frac{\mu_0 nI}{2}\,[\sin\alpha + \sin\beta]$$

Note:

(1) If the solenoid is of infinite length and the point is well inside the solenoid, $\alpha = \beta = (\pi/2)$,

$B = \left(\frac{\mu_0}{4\pi}\right)(2\pi nI)[1+1]$ or $\quad B = \mu_0 nI$.

(2) If the solenoid is of infinite length and the point is near one end, $\alpha = 0$ and $\beta = (\pi/2)$,

$B = \left(\frac{\mu_0}{4\pi}\right)(2\pi nI)[1+0]$ or $\quad B = 1/2\,(\mu_0 nI)$

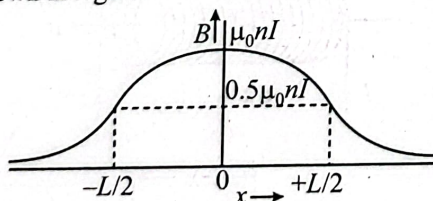(3) If the solenoid is of finite length and the point is on the perpendicular bisector of its axis, $\alpha = \beta$,

$B = \left(\frac{\mu_0}{4\pi}\right)(4\pi nI)\sin\alpha$ with

$\sin\alpha = \dfrac{L}{\sqrt{L^2 + 4R^2}}$ where L is the length of solenoid.

(4) If the solenoid is of finite length and the point is on its axis but near one end, $\beta = 0$,

$B = \left(\frac{\mu_0}{4\pi}\right)(2\pi nI)\sin\alpha$ with $\sin\alpha = \dfrac{L}{\sqrt{L^2 + R^2}}$

(5) The field variation with distance along the axis of a solenoid is as shown in figure.

Magnetic Field Due to Uniformly Moving Charge
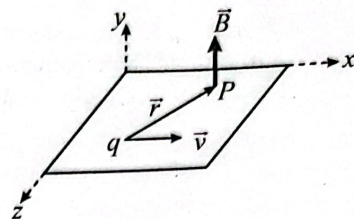
Biot-Savart law gives the magnetic field produced by a current element $I\,d\vec{L}$. However, since $I = dq/dt$, we can write,

$$I\,d\vec{L} = \frac{dq}{dt}\,d\vec{L} = dq\frac{d\vec{L}}{dt} = dq\,\vec{v}$$

$$\therefore \quad d\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{I\,d\vec{L} \times \vec{r}}{r^3} = \left(\frac{\mu_0}{4\pi}\right)dq\frac{\vec{v} \times \vec{r}}{r^3}$$

If a single charge q is moving with a velocity $\vec{v}$, it creates a magnetic field given by

$$\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{q[\vec{v} \times \vec{r}]}{r^3}$$

Train Your Brain

Example 6: Two long solenoids of area of cross section A and 2A are placed coaxially. They both carry same current I in same sense and have same number of turns per unit length 'n'. Then magnetic field on axis is.

Sol. Since, both carry current in same sense, hence magnetic field will be in same direction.

$\therefore \quad B_{net} = \mu_0 nI + \mu_0 nI = 2\mu_0 nI$

Example 7: A solenoid of length 0.2 m has 500 turns on it. If 8.71×10^{-6} Wb/m^2 be the magnetic field at an end of the solenoid, then find the current flowing in the solenoid.

Sol. We know, $B_{end} = \dfrac{\mu_0 ni}{2}$

Here $n = \dfrac{500}{0.2} = 2500$ / metre, $i = ?$, $B_{end} = 8.71 \times 10^{-6}$T

$\therefore \quad i = \dfrac{2B_{end}}{\mu_0 n} = \dfrac{2 \times 8.71 \times 10^{-6}}{4\pi \times 10^{-7} \times 2500}$

$= \dfrac{17.42 \times 10^{-3}}{\pi} = \dfrac{0.01742}{\pi}$A

Example 8: A charge of one coulomb is placed at one end of a non-conducting rod of length 0.6 m. The rod is rotated in a vertical plane about a horizontal axis passing through the other end of the rod with angular frequency $10^4\pi$ rad/s. Find the magnetic field at a point on the axis of rotation at a distance of 0.8 m from the centre of the path.

Sol. As the revolving charge q is equivalent to a current

$$i = qf = q \times \frac{\omega}{2\pi} = 1 \times \frac{10^4\pi}{2\pi} = 5 \times 10^3\text{ A}$$
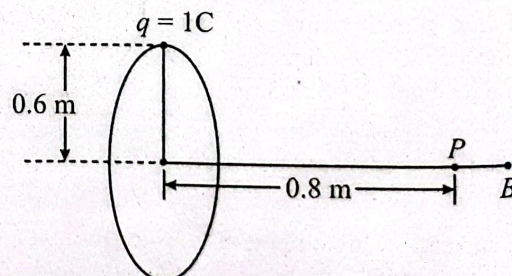
Now $B = \dfrac{\mu_0}{4\pi}\dfrac{2\pi IR^2}{(R^2 + x^2)^{3/2}}$

$\therefore \quad B = 10^{-7} \times \dfrac{2\pi \times 5 \times 10^3 (.6)^2}{[(.6)^2 + (.8)^2]^{3/2}} = 1.13 \times 10^{-3}$ T

6. Magnetic field due to current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the center is 54 μT. What will be its value at the center of the loop?
   - (a) 250 μT
   - (b) 150 μT
   - (c) 125 μT
   - (d) 75 μT

7. A long solenoid has 200 turns per cm and carries a current of 2.5 A. The magnetic field at the center is ($\mu_0 = 4\pi \times 10^{-7}$ Wb/Am)
   - (a) 5.41×10^{-3} Wb/m²
   - (b) 6.28×10^{-2} Wb/m²
   - (c) 2.2×10^{-4} Wb/m²
   - (d) None of these

8. A long solenoid carrying a current produces a magnetic field B along its axis. If the current is doubled and the number of turns per cm is halved, the new value of the magnetic field is:
   - (a) B/2
   - (b) B
   - (c) 2B
   - (d) 4B

AMPERE'S CIRCUITAL LAW

Ampere's circuital law gives another method to calculate the magnetic field due to a given current distribution. It states:

The integral $\oint \vec{B} \cdot d\vec{L}$ of the resultant magnetic field along a closed plane curve is equal to μ_0 times the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant. Thus,

$$\oint \vec{B} \cdot d\vec{L} = \mu_0 I$$

Note:

- The magnetic field $\vec{B}$ on the left-hand side is the resultant field due to all the currents existing anywhere, including I_4 and I_5.
- On the right hand side, the current I is the algebraic sum of all currents passing through the area. Any current outside the area is not included. (Thus, I_4 and I_5 are not included in I.)
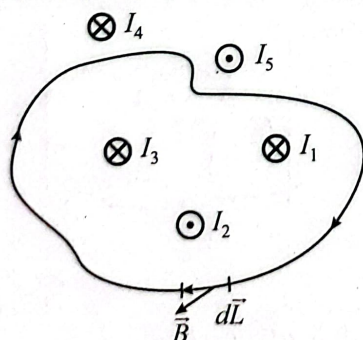
- To find the sign of the enclosed currents, we assign a sense to the curve by putting an arrow on the curve. This becomes the direction of $d\vec{L}$. If you curl the fingers of the right hand along the arrow on the curve, the stretched thumb gives the positive direction of the currents. Thus,
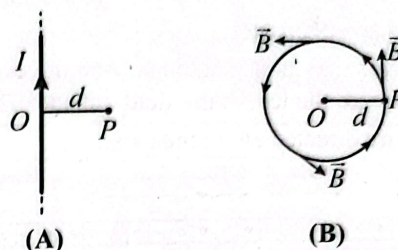
$$I = I_1 + I_3 - I_2$$

- Ampere's law is more useful in finding $\vec{B}$ in certain symmetrical conditions.

APPLICATIONS OF AMPERE'S LAW

Field due to an Infinitely Long Straight Current Carrying Conductor

Consider a circular path passing through point P, with centre at O and radius d.

(A)   **(B)**

We have arbitrarily put the arrow on the circle in anticlockwise direction. The magnetic field is along the tangent, as shown. Same is the direction of the length-element $d\vec{L}$. By symmetry, the magnitude of the magnetic field should be the same at all these points. The circulation of magnetic field along the circle is

$$\oint \vec{B} \cdot d\vec{L} = \oint B \, dL \cos 0° = B \oint dL = B 2\pi d$$

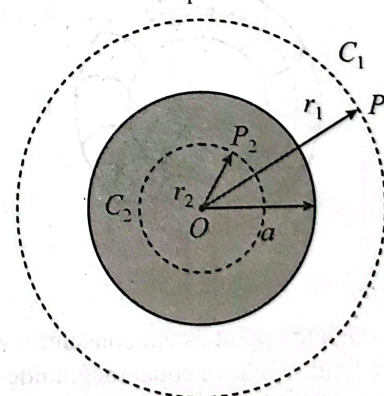The current crossing the area bounded by the circle is I. Thus, from Ampere's law,

$$B 2\pi d = \mu_0 I \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi d}$$

Field Outside and Inside a Thick Long Straight Cylinder

It follows from symmetry that the lines of $\vec{B}$ are in the form of circles with centre at the cylinder axis. For any point P_1, outside the cylinder,

$$\oint \vec{B} \cdot d\vec{L} = \mu_0 I$$

or $\quad B 2\pi r_1 = \mu_0 I \quad$ or $\quad B_{out} = \frac{\mu_0 I}{2\pi r_1}$

Thus, for any external point, the straight current carrying cylindrical conductor behaves as a straight current carrying wire.

Consider a point P_2 lying on the circular contour C_2 of radius r_2 inside the cylinder. The current enveloped by this contour is

$$I_2 = I \left(\frac{\pi r_2^2}{\pi a^2} \right) = I \left(\frac{r_2}{a} \right)^2$$

Where a is the radius of the cylinder
From Ampere's Law,

$$\oint \bar{B} \cdot d\bar{L} = \mu_0 I_2$$

or $B2\pi r_2 = \mu_0 I \left(\dfrac{r_2}{a}\right)^2$ or $B_{in} = \dfrac{\mu_0}{2\pi} \dfrac{I r_2}{a^2}$

Note: If the cylinder has a shape of a tube, the field outside is same as given above, but the field inside the tube is zero, as no current is enclosed by the circular contour.

Field Due to a Solenoid

Let there be n turns per unit length and I be the current flowing. Longer the solenoid, the less is the field outside. The lines of the field $\bar{B}$ inside are directed along the axis.
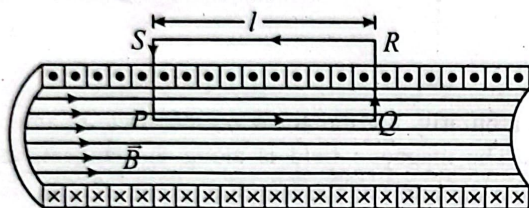
Consider a rectangular contour $PQRS$. The line PQ is parallel to the solenoid axis and hence parallel to the magnetic field $\bar{B}$ inside the solenoid. Thus,

$$\int_P^Q \bar{B} \cdot d\bar{l} = Bl$$

On the remaining three sides, $\bar{B} \cdot d\bar{l}$ is zero everywhere as $\bar{B}$ is either zero (outside the solenoid) or perpendicular to $d\bar{l}$ (inside the solenoid).

Using Ampere's Law,

$$\oint \bar{B} \cdot d\bar{l} = \mu_0 I_{total} \quad \text{or} \quad Bl = \mu_0 (nIl) \quad \text{or} \quad B = \mu_0 n I.$$

Field in a Toroid

If a solenoid is bent in a circular shape and the ends are joined, we get a toroid. Alternatively, one can start with a non-conducting ring or a torus and wind a conducting wire closely on it.
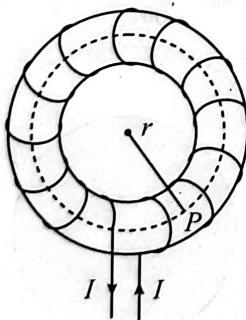
Draw a circle through the point P and concentric with the toroid. By symmetry, the field will have equal magnitude at all points of this circle. Also, the field is everywhere tangential to the circle, Thus,

$$\oint \bar{B} \cdot d\bar{L} = \int B \, dL = B \int dL = 2\pi r B$$

If the total number of turns is n, the current crossing the area bounded by the circle is nI. Using Ampere's law on this circle,

$$\oint \bar{B} \cdot d\bar{L} = \mu_0 n I \quad \text{or} \quad 2\pi r B = \mu_0 n I \quad \text{or} \quad B = \dfrac{\mu_0 n I}{2\pi r}$$

Note: The magnetic field varies as $1/r$ inside a toroid. If the chosen contour passes inside or outside the toroid, it does not envelop any currents, hence $2\pi r B = 0$, i.e., $B = 0$.

Field Due to an Infinite Current Sheet

Consider an infinitely large sheet of current, with λ linear current density (i.e, the current per unit length). Imagine a rectangular loop $abcda$ around the sheet as shown. The current enveloped by this contour is

$I = \lambda L$
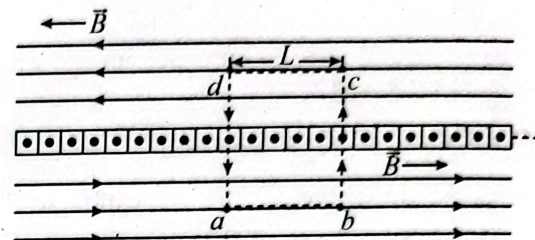
For this symmetry, the magnetic field is uniform on the two sides of the sheet. Applying Ampere's law,

$$\oint \bar{B} \cdot d\bar{L} = \mu_0 I$$

or $\int_a^b \bar{B} \cdot d\bar{L} + \int_b^c \bar{B} \cdot d\bar{L} + \int_c^d \bar{B} \cdot d\bar{L} + \int_d^a \bar{B} \cdot d\bar{L} = \mu_0 \lambda L$

Here, along the paths $b \to c$ and $d \to a$, $\bar{B}$ is perpendicular to $d\bar{L}$. Hence,

$$\int_b^c \bar{B} \cdot d\bar{L} = \int_d^a \bar{B} \cdot d\bar{L} = 0$$

Further, along the paths $a \to b$ and $c \to d$, $\bar{B}$ is parallel to $d\bar{L}$. Hence,

$$\int_a^b \bar{B} \cdot d\bar{L} = \int_c^d \bar{B} \cdot d\bar{L} = BL$$

$\therefore \quad 2BL = \mu_0 \lambda L \quad \text{or} \quad B = \dfrac{\mu_0 \lambda}{2}.$

Train Your Brain

Example 9: Find the value of $\oint \vec{B} \cdot d\vec{\ell}$ for the loops L_1, L_2, L_3 in the figure shown. The sense of $d\vec{\ell}$ is mentioned in the figure.
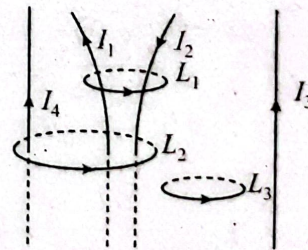
Sol. For L_1 $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 (I_1 - I_2)$

Here I_1 is taken positive because magnetic lines of force produced by I_1 is anti clockwise as seen from top.

I_2 produces lines of $\vec{B}$ in clockwise sense as seen from top. The sense of $d\vec{\ell}$ is anticlockwise as seen from top.

For L_2: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0(I_1 - I_2 + I_4)$

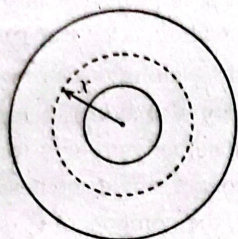For L_3: $\oint \vec{B} \cdot d\vec{\ell} = 0$

Example 10: A long hollow cylindrical wire carries a current I. The internal radius is R and external radius is $2R$. The magnetic field at radial distance x $(R < x < 2R)$

Sol.

Now, according to diagram,

$$\oint \vec{B} \cdot d\vec{l} = B 2\pi x$$

Current through ampere's loop of radius x is i

Hence,

$$i = \frac{I \times (\pi(x)^2 - \pi(R)^2)}{(\pi(2R)^2 - \pi R^2)}$$

$$i = \frac{I(x^2 - R^2)}{3R^2}$$

By Ampere's law,

$$B 2\pi x = \frac{\mu_0 I(x^2 - R^2)}{3R^2}$$

$$B = \frac{\mu_0 I(x^2 - R^2)}{6\pi R^2 x}$$

Concept Application

9. A long solid cylindrical wire with radius of cross section 'R' has current density $J = 3x$ A/m² (where x is radial distance). Find magnetic field at $x > R$

(a) $\dfrac{\mu_0 R^2}{x}$ (b) $\dfrac{\mu_0 R^3}{x}$

(c) $\dfrac{\mu_0 x^2}{R}$ (d) $\dfrac{2\mu_0 x^2}{R}$

10. A long hollow cylindrical wire of internal radius R external radius $2R$ carries a uniformly distributed current I. Another long solid cylindrical wire of radius R carries equal current I. The ratio of magnetic field at $x = 3R$ from both wire is (x is radial distance)

(a) 1   (b) 2

(c) 3   (d) 4

11. A current of $1/4\pi$ ampere is flowing through a toroid. It has 1000 number of turn per meter. Then value of magnetic field (in Wb/m²) along its axis is :

(a) 10^{-2} (b) 10^{-3}

(c) 10^{-4} (d) 10^{-7}

12. A long straight wire of radius a carries a steady current I. The current is uniformly distributed across its cross-section. Find the ratio of the magnetic field at $\dfrac{a}{2}$ and $2a$.

(a) 1   (b) 2

(c) 3   (d) 4

MAGNETIC FORCE ON MOVING CHARGES

An electric charge at rest, kept in a magnetic field, experiences no force. Magnetic field acts only on moving charges.

The force exerted by a magnetic field $\vec{B}$ on a charged particle q moving with a velocity $\vec{v}$ is given by

$$\vec{F} = q(\vec{v} \times \vec{B})$$

The magnitude of this force is given as

$$F = qvB \sin\theta$$

where θ is the angle between velocity vector $\vec{v}$ and the magnetic field vector $\vec{B}$.

The direction of the force is same as the cross-product. It is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$, and is given by the right-hand rule.

If the vector $\vec{v}$ and $\vec{B}$ are specified in a three-dimensional space in terms of their components along x, y and z-axis as

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

Then the magnetic force $\vec{F}$ is given as

$$\vec{F} = q\,(\vec{v} \times \vec{B}) = q \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= q\,\hat{i}(v_yB_z - v_zB_y) - q\,\hat{j}(v_xB_z - v_zB_x) + q\hat{k}(v_xB_y - v_yB_x)$$

Note: Carefully that

(1) The force on a positive charge is in the direction of $\vec{v} \times \vec{B}$, but the force on a negative charge is opposite to the vector $\vec{v} \times \vec{B}$.
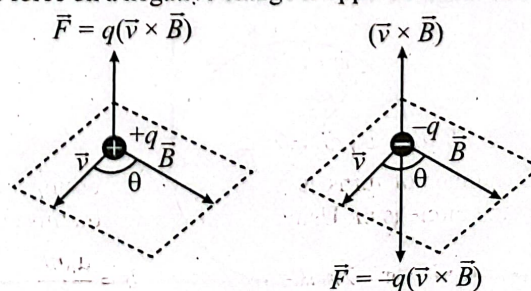
(2) There can be any angle between the vectors $\vec{v}$ and $\vec{B}$, but the resulting force $\vec{F}$ on the charge is always perpendicular to $\vec{v}$ (as well as to $\vec{B}$).

(3) If the motion of the charged particle is collinear with the field, i.e., $\theta = 0°$ or $180°$, $F = qvB \sin \theta = 0$

Thus, a moving charged particle does not experience any force in a magnetic field if its motion is parallel or antiparallel to the field.

DIFFERENCE BETWEEN MAGNETIC FORCE AND ELECTRIC FORCE

(1) Magnetic force is always perpendicular to the field, whereas electric force is collinear with the field.

(2) Magnetic force is velocity dependent, i.e., acts only when the charged particle is in motion, whereas electric force (qE) is independent of the state of rest or motion of the charged particle.

(3) Work done by magnetic force is zero when the charged particle is displaced (since magnetic force is perpendicular to velocity), whereas electric force might do work in displacing the charged particle.

(4) Magnetic force never acts on a charged particle at rest. (as $v = 0$)

(5) Magnetic force never act on uncharged (neutral) particle (as $q = 0$)

MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

Case I: Straight Line Motion

If a charged particle q is projected into a uniform magnetic field $\vec{B}$ with a velocity $\vec{v}$ which is parallel to the field lines, the force experienced by the charge is zero. Hence, it travels in a straight line with uniform velocity.

Case II: Uniform Circular Motion

If a charged particle q is projected into a uniform magnetic field $\vec{B}$ with an initial velocity $\vec{v}$ perpendicular to the magnetic field, it gets trapped in a circular path. The force exerted by the field provides the necessary centripetal force, $qvB = \dfrac{mv^2}{r}$,

where r is the radius of the circular path. Thus, we have $r = \dfrac{mv}{qB}$

The radius of the orbit is directly proportional to the linear momentum of the particle and inversely proportional to the magnitude of magnetic field.
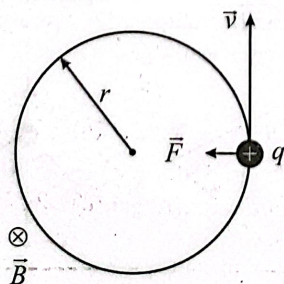
The time period of the orbit is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

Time period is independent of velocity. All particles with same charge-to-mass ratio (q/m), have the same time period in a given magnetic field.

Case III: Motion in Helical Path

When a charged particle is projected into a uniform magnetic field at an angle, it travels in a helical path. The figure shows uniform magnetic field $\vec{B}$ along positive x-direction. A charge q is projected with velocity $\vec{v}$ making an angle θ with x-axis. The two components of the velocity are $v_x = v \cos \theta$ and $v_y = v \sin \theta$. The component v_x is along the field and hence remains unchanged by the field. However, the component v_y is perpendicular to the field, and the particle describes a circle (in a plane perpendicular to the field, i.e., in y-z plane) of radius,

$$r = \frac{m(v \sin \theta)}{qB},$$

and time period

$$T = \frac{2\pi r}{v \sin \theta} = \frac{2\pi m}{qB}$$
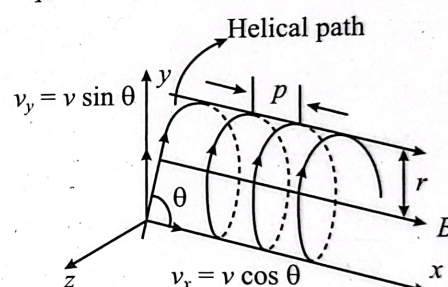
The projection of the helical path onto y-z plane will be a circle.

Whereas the projections onto the x-y plane or the x-z plane will be sinusoidal. The pitch of the helix is defined as the linear distance moved by the charged particle along the magnetic field in one revolution, and is given as

$$p = (v \cos \theta) T = \frac{2\pi m}{qB} v \cos \theta$$

Train Your Brain

Example 11: A 64 µC charge traveling with velocity $\vec{v} = (4\hat{i} + 2\hat{j})$ m/s enters a region of magnetic field of induction $\vec{B} = (-5\hat{i} + 3\hat{j})$T. Compute the force experienced by the charge.

**Sol.** Both $\vec{v}$ and $\vec{B}$ lie in the x-y plane. Therefore, the magnetic force $\vec{F}$ must be along the z-axis. Let us compute $\vec{F}$ using determinant method,

$$\vec{F} = q(\vec{v} \times \vec{B}) = (64 \times 10^{-6}) \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 2 & 0 \\ -5 & 3 & 0 \end{vmatrix}$$

$$= (64 \times 10^{-6})[\hat{i}(0) - \hat{j}(0) + \hat{k}(12 + 10)]$$
$$= (1.41 \times 10^{-3}\hat{k}) \text{ N}$$

Example 12: An electron with a kinetic energy of 10^3 eV moves perpendicular to the direction of a uniform magnetic field, $B = 10^{-4}$ T.

(i) What is the period of its orbit?

(ii) What is the radius of the orbit?

Sol. (i) Time period is given by

$$T = \frac{2\pi m}{eB} = \frac{2\pi(9.1 \times 10^{-31})}{(1.6 \times 10^{-19})(10^{-4})}$$

or $T = 3.6 \times 10^{-7}$ s

(ii) Radius of the orbit is given by

$$r = \frac{mv}{eB} = \frac{p}{eB}$$

The momentum of the electron can be obtained from kinetic energy as

$$p = mv = \sqrt{2Km} = \sqrt{2(1.6 \times 10^{-16})(9.1 \times 10^{-31})}$$
$$= 1.7 \times 10^{-23} \text{ kg m/s}$$

$$\therefore \quad r = \frac{(1.7 \times 10^{-23})}{(1.6 \times 10^{-19})(10^{-4})} = 1.07 \text{ m}$$

Example 13: An α-particle is accelerated by a potential difference of 10^4 V. Find the change in its direction of motion if it enters normally in a region of thickness 0.1 m having transverse magnetic induction of 0.1 T [Mass of α-particle $= 6.4 \times 10^{-27}$ kg].
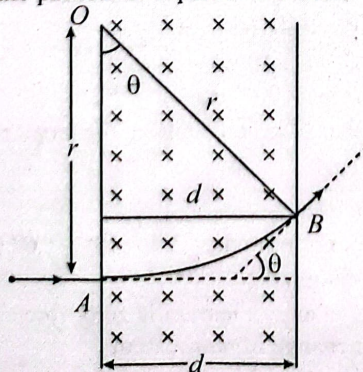
Sol. When an α-particle is accelerated by 10^4 volts, its kinetic energy will be $K = (2e)(10^4 \text{ V}) = 2 \times 10^4$ eV.

The radius of the circular path is

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

$$= \frac{(2 \times 6.4 \times 10^{-27} \times 2 \times 10^4 \times 1.6 \times 10^{-19})^{1/2}}{2 \times 1.6 \times 10^{-19} \times 0.1} = 0.2 \text{ m}$$

Now, the angle between tangents at two points will be equal to the angle between OA and OB. Hence the change in the direction of the particle as it passes the field,

$$\theta = \sin^{-1}\left(\frac{d}{r}\right) = \sin^{-1}\left(\frac{0.1}{0.2}\right) = 30°$$

Example 14: A beam of protons with a velocity of 4×10^5 m/s enters a uniform magnetic field of 0.3 tesla at an angle of 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of the helix.

Sol. Component of the velocity perpendicular to the direction of the field,

$v_\perp = v \sin\theta = 4 \times 10^5 \sin 60° = 2\sqrt{3} \times 10^5$ m/s

Component of the velocity parallel to the field,

$v_\parallel = v \cos\theta = 2 \times 10^5$ m/s

The radius of the helix is given by

$$r = \frac{mv_\perp}{qB} = \frac{1.67 \times 10^{-27} \times 2\sqrt{3} \times 10^5}{0.3 \times 1.6 \times 10^{-19}} = 0.012 \text{ m}$$
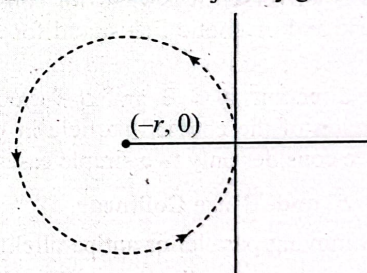
The time period of revolution is

$$T = \frac{2\pi m}{qB} = \frac{2 \times 3.14 \times 1.67 \times 10^{-27}}{0.3 \times 1.6 \times 10^{-19}} = 2.185 \times 10^{-7} \text{ s}$$

Pitch of the helix $= T \times v_\parallel$
$= 2.185 \times 10^{-7} \times 2 \times 10^5 = 4.37 \times 10^{-2}$ m

Example 15: A particle of mass 'm' and charge 'q' is thrown from origin with velocity $v\hat{j}$. A uniform magnetic field exist in the region $\vec{B} = B_0(-\hat{k})$. The equation of trajectory of particle is

Sol. The particle will follow trajectory given below.

$(-r, 0)$

General eq of circle will be $\{x - (-r)\}^2 + y^2 = r^2$ where $r = mv/qB_0$

![Concept Application icon] **Concept Application**

13. A charged particle with charge q and mass 'm' is thrown into space having only magnetic field $\vec{B} = B_0\hat{i}$ with a velocity $\vec{v} = v_0\hat{i} + v_0\hat{j}$. Find the distance travelled by particle in time $t = \dfrac{2\pi m}{qB}$.

(a) zero

(b) $\dfrac{2\pi m v_0}{qB}$

(c) $\dfrac{2mv_0}{qB}$

(d) $2\sqrt{2}\dfrac{\pi m v_0}{qB}$

14. If a proton and deuteron and α-particle on being accelerated by the same potential difference enters perpendicular to the magnetic field, then the ratio of their kinetic energies is
    (a) 1 : 2 : 2   (b) 2 : 2 : 1
    (c) 1 : 2 : 1   (d) 1 : 1 : 2

15. A proton of mass "m" and charge "e" is fired parallel to a uniform magnetic field of induction "B". The radius of curvature of the path of the particle in the field is
    (a) Be/mv (b) mv/Be
    (c) Zero   (d) Infinity

16. A 2 MeV proton is moving perpendicular to a uniform magnetic field of 2.5 tesla. The force on the proton is
    (a) 2.5×10^{-10} newton   (b) 8×10^{-11} newton
    (c) 2.5×10^{-11} newton   (d) 8×10^{-13} newton

LORENTZ FORCE

When a moving charged particle is subjected simultaneously to both electric field $\vec{E}$ and magnetic field $\vec{B}$, the moving charged particle will experience electric force $\vec{F}_e = q\vec{E}$ and magnetic force $\vec{F}_m = q(\vec{v} \times \vec{B})$ Therefore, net force on it will be

$$\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$$

which is called the **Lorentz-force equation**.

Lorentz equation is universal. It is valid for constant as well as for varying electric and magnetic fields, and for any velocity $\vec{v}$ of the charge q.

Depending on the directions of $\vec{v}, \vec{E}$ and $\vec{B}$ various situations are possible. The motion of the charged particle in general is quite complex. Here, we consider only two simple cases:

Case I: When $\vec{v}, \vec{E}$ and $\vec{B}$ are Collinear

As the particle is moving parallel or antiparallel to the magnetic field, the magnetic force on it will be zero. The electric force $\vec{F}_e$ will produce an acceleration,

$$\vec{a} = \frac{\vec{F}_e}{m} = \frac{q\vec{E}}{m}$$

The particle follows a straight line path with change in speed. So in this situation speed, velocity, momentum and kinetic energy all will change, without change in direction, of motion as shown in figure.
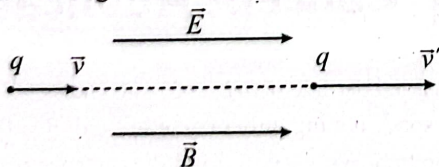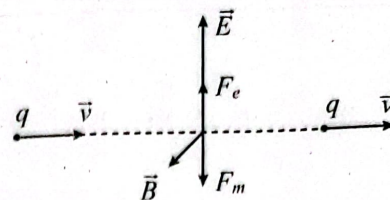

Case II: $\vec{v}, \vec{E}$ and $\vec{B}$ are Mutually Perpendicular

In this situation we consider a specific case when $\vec{E}$ and $\vec{B}$ are such that

$$\vec{F} = \vec{F}_e + \vec{F}_m = 0 \Rightarrow \vec{a} = (\vec{F}/m) = 0$$

The particle will pass through the field with same velocity. And in this situation, as

$F_e = F_m$ or $qE = qvB$ ∴ $v = E/B$

Train Your Brain

Example 16: An electron (mass = 9.1×10^{-31} kg; charge = 1.6×10^{-19} C) experiences no deflection if subjected to an electric field of 6.4×10^5 V/m, and a magnetic field of 2.0×10^{-3} Wb/m². Both the fields are normal to the path of electron and to each other. If the electric field is removed, then what will be the radius of orbit in which electron will orbit?

Sol. For no deflection of electron, $v = \dfrac{E}{B}$

Thus, $v = \dfrac{6.4 \times 10^5}{2 \times 10^{-3}} = 3.2 \times 10^8$ m/s

So, the radius of orbit after electric field is switched off is

$$R = \frac{mv}{qB} = \frac{9.1 \times 10^{-31} \times 3.2 \times 10^8}{1.6 \times 10^{-19} \times 2 \times 10^{-3}} = 0.90 \text{ m}$$

Example 17: A charge particle having charge 4 coulomb is thrown with velocity $2\hat{i} + 3\hat{j}$ inside a region having $\vec{E} = 2\hat{j}$ and magnetic field $5\hat{k}$. Find the initial Lorentz force acting on the particle

Sol. Lorentz force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

$\vec{F} = 4[2\hat{j} + (2\hat{i} + 3\hat{j}) \times 5\hat{k}]$

$= 4[2\hat{j} + 10(-\hat{j}) + 15\hat{i}] = 60\hat{i} - 32\hat{j}$

Example 18: A proton beam passes without deviation through a region of space where there are uniform transverse mutually perpendicular electric and magnetic fields with $E = 120 \dfrac{\text{kV}}{\text{m}}$ and $B = 50$ mT. Then the beam strikes a grounded target. Find the force with which the beam acts on the target if the beam current is equal to $I = 0.80$ mA.

Sol. For no deviation

$eE = evB$

$\Rightarrow v = \dfrac{E}{B} = \dfrac{1.2 \times 10^5}{5 \times 10^{-2}} = 2.4 \times 10^6$ m/s

Beam current = 0.80 mA

∴ Number of protons hitting the target per second

$= \dfrac{0.80 \times 10^{-3}}{1.6 \times 10^{-19}} = 5 \times 10^{15}$

Momentum of protons = $2.4 \times 10^6 \times 1.673 \times 10^{-27}$

$= 4 \times 10^{-21}$ kg m/s

Since the final momentum is zero, force on the target
= Rate of change of momentum
$= 5 \times 10^{15} \times 4 \times 10^{-21} = 2 \times 10^{-5}$ N.

17. An electron enters a region where magnetic (B) and electric (E) fields are mutually perpendicular to one another, then
    (a) It will always move in the direction of B
    (b) It will always move in the direction of E
    (c) It always possess circular motion
    (d) It can go undeflected also

18. A particle of charge -16×10^{-18} coulomb moving with velocity 10 ms^{-1} along the x-axis enters a region where a magnetic field of induction B is along the y-axis, and an electric field of magnitude 10^4 V/m is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is
    (a) 10^{-3} Wb/m^2 (b) 10^3 Wb/m^2
    (c) 10^5 Wb/m^2 (d) 10^{16} Wb/m^2

FORCE ON A CURRENT CARRYING CONDUCTOR IN A MAGNETIC FIELD

A conductor has enormous number of free electrons, the negatively charged particles. When current flows through a conductor, these moving electrons experience a magnetic force which is then transmitted to the conductor.

Thus, the basis of a current carrying conductor experiencing a force when kept in a magnetic field is the Lorentz force on moving charges.

When a current element $Id\vec{L}$ is placed in a magnetic field $\vec{B}$, it experience a force,

$$d\vec{F} = Id\vec{L} \times \vec{B}$$

The **magnitude** of this force is $dF = BI\,dL \sin\theta$

(i) The force will be minimum ($= 0$) when $\sin\theta = 0$, i.e., $\theta = 0°$ or $180°$. That is a current element in a magnetic field does not experience any force if the current in it is collinear with the field.
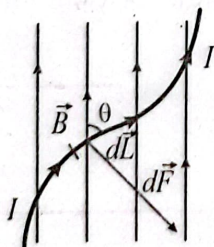
(ii) The force will be maximum when $\sin\theta = 1$, i.e., $\theta = 90°$. That is, the force on a current element in a magnetic field is maximum ($= BIdL$) when it is perpendicular to the field.

(iii) The direction of the force is given by the cross product $Id\vec{L} \times \vec{B}$.

Important Points

(i) The force $BIdL\sin\theta$ is not a function of position r. That is, the magnetic force on a current element is non-central. (A central force is of the form $F = Kf(r)\,\hat{n}_r$)

(ii) The force $d\vec{F}$ is always perpendicular to both $\vec{B}$ and $Id\vec{L}$, though $\vec{B}$ and $Id\vec{L}$ may or may not be perpendicular to each other.

(iii) If the magnetic field is uniform and the current is constant, both can be taken out from the integral,

$$\vec{F} = \int d\vec{F} = \int I\,d\vec{L} \times \vec{B} = I\left\{\int d\vec{L}\right\} \times \vec{B}$$

The integral $\int d\vec{L}$ represents the vector sum of all the length elements from initial point (A) to final point (B). It shows that the force on a curved current carrying wire of any arbitrary shape between points A and B kept in a uniform magnetic field, is same as that on a straight current carrying wire joining these points.
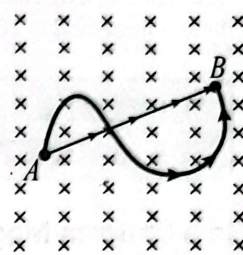
FORCE OF INTERACTION BETWEEN TWO LONG PARALLEL CURRENT CARRYING WIRES

The net force on a current carrying conductor due to its own field is zero. However, if there are two long parallel current carrying wires 1 and 2 as shown, wire 1 will be in the field of wire 2 and vice-versa. So the force on dL_2 length of wire 2 due to field of wire 1,

$$dF_2 = I_2 dL_2 B_1 = I_2 dL_2 \left[\left(\frac{\mu_0}{4\pi}\right)\frac{2I_1}{d}\right] = \left(\frac{\mu_0}{4\pi}\right)\frac{2I_1 I_2}{d} dL_2$$
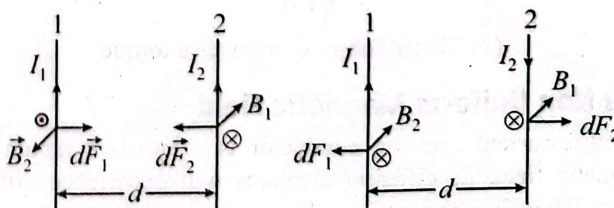
Therefore, the force per unit length is

$$\frac{dF_2}{dL_2} = \left(\frac{\mu_0}{4\pi}\right)\frac{2I_1 I_2}{d}$$

Same will be true for wire-1 in the field of wire-2,

$$\frac{dF_1}{dL_1} = \left(\frac{\mu_0}{4\pi}\right)\frac{2I_1 I_2}{d}$$

The two forces are seen to be equal in magnitude. Therefore, the interaction force per unit length in case of two parallel current carrying wires separated by a distance d is given by

$$\frac{dF}{dL} = \left(\frac{\mu_0}{4\pi}\right)\frac{2I_1 I_2}{d}$$

(A)   (B)

The direction of the force can be determined in accordance with the rules specified. It is found that the force between the wires is attractive if the currents in them are in the same direction, otherwise repulsive (this is **opposite** to that of what happens between two charges).

SI Unit of Current

The SI unit of current **ampere**, which is one of the base units is defined using the above concept as follows:

One ampere of current is the current which when passing through each of two parallel infinitely long straight conductors placed in free space at a distance of 1 m from each other produces between them a force of 2×10^{-7} newton for one metre of their length.

CURRENT CARRYING LOOP IN A MAGNETIC FIELD

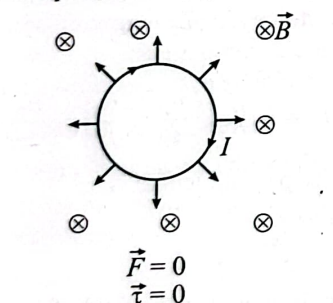### In a Uniform Magnetic Field

If the current carrying loop of any arbitrary shape is placed in a uniform magnetic field, the net force is

$$\vec{F} = \oint d\vec{F} = \oint I d\vec{L} \times \vec{B} = I \left\{ \oint d\vec{L} \right\} \times \vec{B} = 0$$
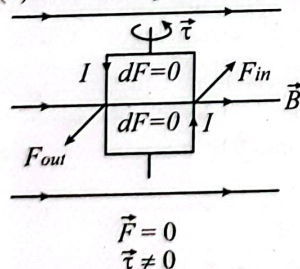
(as the vector sum of $d\vec{L}$ for a closed loop is always zero)

Thus, the net magnetic force on current loop in a uniform magnetic field is always zero.

Note that different parts of the loop may experience elemental force due to which the loop may be under tension (Figure (a)) or may experience a torque (Figure (b)).

$$\vec{F} = 0$$
$$\vec{\tau} = 0$$

(a) Closed loop under tension

$$\vec{F} = 0$$
$$\vec{\tau} \neq 0$$

(b) Closed loop experiences a torque

In a Non Uniform Magnetic Field

When a current carrying conductor is situated in nonuniform magnetic field, its different elements will experience different forces. Therefore, the net force F_R is not zero. However, the torque τ acting on a loop may or may not be zero.

Consider a current carrying rod CD placed perpendicular to a long current carrying conductor AB. The element of the rod nearer to the conductor AB will experience larger force dF, as shown. Obviously, the rod experience a net force and also a net torque. If free to move, the root CD will translate as well as rotate to a new position $C'D'$. In this position, the torque τ becomes zero and the rod experiences a net repulsive force.
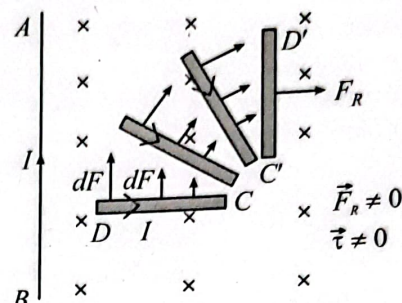
Next consider a rectangular loop $PQRS$ placed in the magnetic field produced by a long current carrying conductor AB. The elements of sides PQ and RS experience different forces, as shown. The forces acting on the side PQ are equal and opposite to the forces acting on side RS. Hence, the loop experiences no net force in vertical direction.
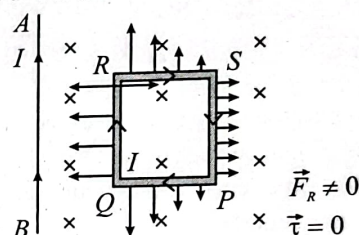

The side QR experiences an attractive force and the side SP experiences a repulsive force. However, as QR is nearer to AB as compared to SP, the attractive force is large than the repulsive force. Hence the loop experience a net attractive force F_R. Note that no torque is acting on the loop.

Train Your Brain

Example 19: A rigid square loop of side a carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. Find the net force on the loop due to wire.
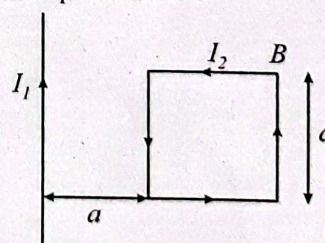
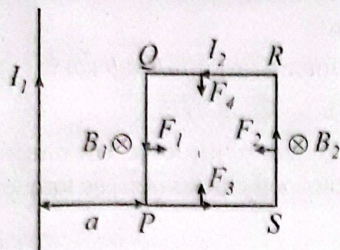**Sol.** F_3 and F_4 cancel each other (From symmetry)
Force on PQ will be $F_1 = I_2 B_1 a$

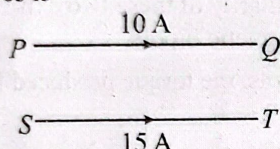$$= I_2 \frac{\mu_0 I_1}{2\pi a} a = \frac{\mu_0 I_1 I_2}{2\pi}$$

Force on RS will be $F_2 = I_2 B_2 a = I_2 \left(\dfrac{\mu_0 I_1}{2\pi(2a)} \right) a$

$= \dfrac{\mu_0 I_2 I_1}{4\pi}$

$F_{net} = \dfrac{\mu_0 I_1 I_2}{4\pi}$ (repulsion)

Example 20: In the adjoining figure the two parallel wires PQ and ST are at 30 cm apart. The currents flowing in the wires are according to figure. The force acting over a length of 5 m of the wires is

$P \xrightarrow{\quad 10\ A \quad} Q$

$S \xrightarrow{\quad 15\ A \quad} T$

Sol. When currents flow in two long, parallel wires in the same direction, the wires exert a force of attraction on each other. The magnitude of this force acting per meter length of the wires is given by

$F = \dfrac{\mu_0}{2\pi} \dfrac{i_1 i_2}{R} = 2 \times 10^{-7} \dfrac{i_1 i_2}{R}$ N/m

Here $i_1 = 10$ A, $i_2 = 15$A,

$R = 30$ cm $= 0.3$ m

$\therefore \quad F = 2 \times 10^{-7} \dfrac{10 \times 15}{0.3} = 1 \times 10^{-4}$ N/m

$\therefore$ Force on 5 m length of the wire

$= 5 \times (1 \times 10^{-4}) = (5 \times 10^{-4}) = 5 \times 10^{-4}$ N (attraction)

Concept Application

19. A closed loop is placed in a uniform magnetic field. If the force experienced by the loop is F, then

   (a) $F = 0$, always.
   (b) $F = 0$, only it B is perpendicular to loop.
   (c) $F = 0$, only if B is parallel to loop.
   (d) F may be equal to zero.

20. A horizontal magnetic field at a certain place is 3.0×10^{-5} T and has a direction from the south to north. The force per unit length on a very long straight carrying a steady current of 1.2A in east to west direction is:

   (a) 3.0×10^{-5} N m^{-1} (b) 3.2×10^{-5} N m^{-1}
   (c) 3.6×10^{-5} N m^{-1} (d) 3.8×10^{-5} N m^{-1}

MAGNETIC DIPOLE MOMENT

Magnetic field lines due to a solenoid are very similar to those of a bar magnet. A bar magnet (or a solenoid) can be thought of as a 'dipole'. Its field lines resemble very close to those of an electric dipole. This connection is made more precise by associating a magnetic moment with a current loop .

Current Loop as Magnetic Dipole

The magnetic moment $\vec{p}_m$ associated with a current loop carrying current I and of area A is defined as

$\vec{p}_m = IA\hat{n}$

Where $\hat{n}$ is a unit vector normal to the loop, whose direction is connected with the direction of current in the loop through the **right-hand screw rule**. If the current is directed along the four curved fingers of the right-hand, $\hat{n}$ (and hence $\vec{p}_m$) is directed along the stretched out thumb.

Note:

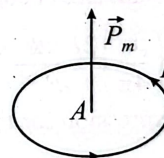(1) Magnetic moment is a vector, with magnitude,

   $p_m = IA$

(2) Its SI unit is A m^2.

(3) Its dimensions are $[L^2A]$.

(4) Its value depends on the current in the loop and its area. It is independent of the shape of the loop, whether circular or rectangular or any other arbitrary shape.

(5) If the loop (or the coil) contains N number of turns,

   $p_m = NIA$

Charged Particle Moving in a Circle

Consider a particle of mass m and having charge q moving with a velocity $\vec{v}$ in a circle of radius R. The associated current is
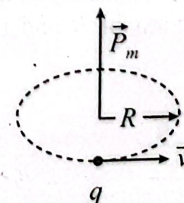
$I = \dfrac{q}{T} = \dfrac{q\,v}{2\pi R}$

Area of the circular path is

$A = \pi R^2$

$\therefore \quad \vec{p}_m = IA\hat{n} = \dfrac{qv}{2\pi R} \pi R^2 \hat{n} = \dfrac{1}{2} qvR\hat{n}$

But the angular momentum of this particle is

$\vec{L} = mvR\hat{n}$

$\therefore \quad \vec{p}_m = \dfrac{q}{2m} \vec{L}$

Thus, the magnetic moment of a charged particle moving in a circle is $(q/2m)$ times its angular momentum. Note that if the charge on the particle is negative (e.g., an electron), the $\vec{p}_m$ is directed opposite to $\vec{L}$.

Field Pattern Due to Circular Coil

A circular coil carrying current behaves like a **thin disc magnet**. One face of this magnet is north pole and the other is south pole. Just look at one face of the coil. If the current around the face is in anticlockwise direction, it is a north pole (see figure). On the other hand, if the current is in clockwise direction, then the face is a south pole (see figure.) Note that the anticlockwise direction of current coincides with the arrowheads at the ends of the letter N; and the clockwise direction of current coincides with the arrowheads at the ends of the letter S.

| (a) Perspective view | (b) View from side A | (c) View from side B |

Magnetic Field Due to a Magnetic Dipole

Magnetic field due to a current carrying coil of radius R, at an axial point at a distance d is given as

$$B = \left(\frac{\mu_0}{4\pi}\right) \frac{IR}{(R^2 + d^2)^{3/2}} 2\pi R$$

If $d \gg R$, i.e., for very distant axial point,

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{IR}{d^3} 2\pi R = \left(\frac{\mu_0}{4\pi}\right)\frac{2I\,\pi R^2}{d^3} \quad \text{or} \quad B = \frac{\mu_0}{4\pi}\frac{2p_m}{d^3}$$

Compare this, with the expression for the electric field due to an electric dipole,

$$E = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2p_e}{d^3}$$

The remarkable similarity of the above two expressions justify associating a magnetic dipole with a small current loop.

Torque on a Magnetic Dipole

Just as an electric dipole placed in an electric field experiences a torque, similarly a magnetic dipole (a current loop) placed in a magnetic field experiences a torque.

Consider a rectangular loop of sides a and b, carrying a current I. Let it be placed in a uniform magnetic field B. Let the angle between the direction of B and the perpendicular to the plane of the coil be θ.

| (a) Side View | (b) Top View |

The element PQ experiences an upward force. The element RS experiences an equal force downward. So, the net force and torque due to these are zero. As shown in figure (b), the force on SP is equal and opposite to the force on QR. Each of these forces has magnitude IaB. Since these are not along the same line, a net torque is produced,

$$\tau = \text{force} \times (\text{force-arm}) = IaB \times b \sin\theta$$

$$\text{or} \quad \tau = IabB \sin\theta = (IS \sin\theta)\, B$$

where $A = ab$ is the area of the loop. The direction of the torque is such as to rotate the loop clockwise. The torque is fully defined by

$$\vec{\tau} = \vec{p}_m \times \vec{B}$$

where $p_m = IA$ is the magnetic moment having a direction perpendicular to the plane of the loop.

The torque on an electric dipole (of dipole moment $\vec{p}_e$) placed in an electric field $\vec{E}$ is given as

$$\vec{\tau} = \vec{p}_e \times \vec{E}$$

Once again, the similarity of these two expressions confirm that a current loop is a magnetic dipole.

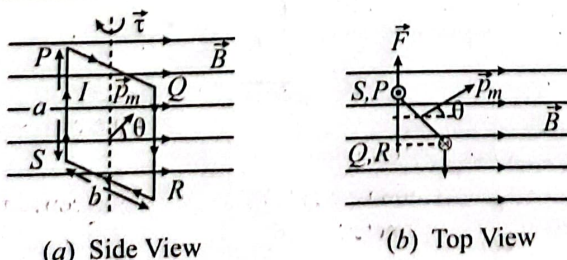If the loop has N turns, the torque produced becomes

$$\tau = NIBA \sin\theta$$

Potential Energy of a Current Loop

When a bar magnet is freely suspended in Earth's magnetic field, it aligns itself such that its north pole points towards geographical north. Similarly, when a current loop is suspended in a uniform magnetic field, it rotates to attain equilibrium position, in which it lies with its vector magnetic moment $\vec{p}_m$ in the same direction as the field $\vec{B}$.

Consider a rectangular loop $ABCD$, carrying current I placed in a magnetic field B along z-axis. The loop can rotate about an axis parallel to x-axis, and passing through the mid-points of the sides AB and CD. If the normal to loop makes an angle θ with the z-axis, the torque acting on the loop is

$$\tau = -p_m B \sin\theta$$
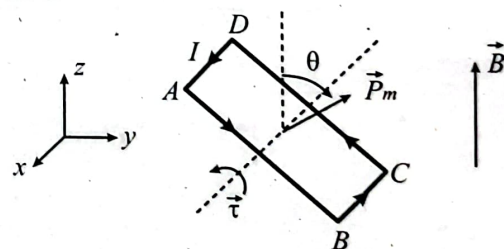
The extra minus sign is included here because the torque is in the direction of decreasing θ.

Work done dW in rotating the dipole through a small angle $d\theta$ against this torque is

$$dW = \tau\, d\theta = -p_m B \sin\theta\, d\theta$$

Therefore, the work done in rotating the dipole from angular position θ_1 to θ_2,

$$W = -\int_{\theta_2}^{\theta_1} p_m B \sin\theta\, d\theta = p_m B [\cos\theta_1 - \cos\theta_2]$$

If the dipole is rotated from field direction, i.e., $\theta_1 = 0$ to any position θ, i.e., θ_2, we have

$$W = p_m B [1 - \cos\theta]$$

For a dipole placed in a magnetic field, the potential energy U of the dipole is defined as the work done in rotating the dipole from a direction perpendicular to the field [Figure (b)] to the given direction, i.e.,

$$U = W(\theta) - W(90°)$$

$$= p_m B (1 - \cos\theta) - p_m B$$

$$= -p_m B \cos\theta$$

or $U = -\vec{p}_m \cdot \vec{B}$

This is similar to the potential energy of an electric dipole moment $\vec{p}_e$ placed in an electric field $\vec{E}$,

$$U = -\vec{p}_e \cdot \vec{E}$$

The potential energy of magnetic dipole is minimum ($= -p_m B$) when $\vec{p}_m$ and $\vec{B}$ are parallel [Figure (a)]. This is the position of **stable equilibrium**. In this position, $\tau = 0$. When $\vec{P}_m$ and $\vec{B}$ are antiparallel [Figure (c)], again $\tau = 0$, work done is $2p_m B$ and the potential energy is $p_m B$. This is the maximum potential energy the current loop can have. It is in **unstable equilibrium**.

(a)   (b)   (c)
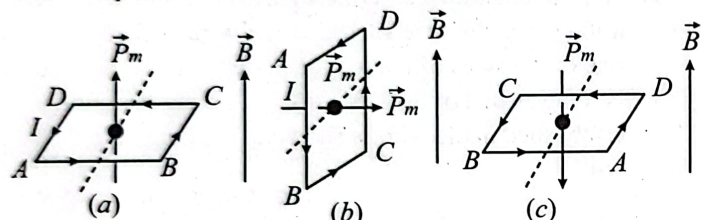
$\theta = 0°\,(\vec{p}_m \parallel \vec{B})$ $\theta = 90°\,(\vec{P}_m \perp \vec{B})$ $\theta = 180°\,(\vec{P}_m \text{ anti} \parallel \vec{B})$

$\tau = 0$ (min) $\tau = p_m B$ (max) $\tau = 0$ (min)

$W = 0$ (min) $W = p_m B$ $W = 2p_m B$ (max)

$U = -p_m B$ (min) $U = 0$ $U = p_m B$ (max)

Stable equilibrium   No equilibrium   Unstable equilibrium

Comparison between Electric and Magnetic Dipoles

S.No.	Property	Electric Dipole	Magnetic Dipole
1.	Definition	$p_e = 2qa$	$p_m = IA$
2.	Field at a distant point along axis (end-on position)	$E_{\parallel} = \left(\dfrac{1}{4\pi\varepsilon_0}\right)\dfrac{2p_e}{d^3}$	$B_{\parallel} = \left(\dfrac{\mu_0}{4\pi}\right)\dfrac{2p_m}{d^3}$
3.	Field at a distant point along perpendicular bisector (broad-side-on position)	$E_{\perp} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{p_e}{d^3}$	$B_{\perp} = \left(\dfrac{\mu_0}{4\pi}\right)\dfrac{p_m}{d^3}$

S.No.	Property	Electric Dipole	Magnetic Dipole
4.	Torque in an external field	$\vec{\tau} = \vec{p}_e \times \vec{E}$	$\vec{\tau} = \vec{p}_m \times \vec{B}$
5.	Work done in rotating the dipole in an external field from its equilibrium position	$W = p_e E$ $(1 - \cos\theta)$ $= p_e E - \vec{p}_e \cdot \vec{E}$	$W = p_m B$ $(1 - \cos\theta)$ $= p_m B - \vec{p}_m \cdot \vec{B}$
6.	Potential energy in an external field	$U = -\vec{p}_e \cdot \vec{E}$	$U = -\vec{p}_m \cdot \vec{B}$

MOVING COIL GALVANOMETER

The torque acting on a current loop placed in a magnetic field forms the basis of working of a galvanometer (and also of electric motor).
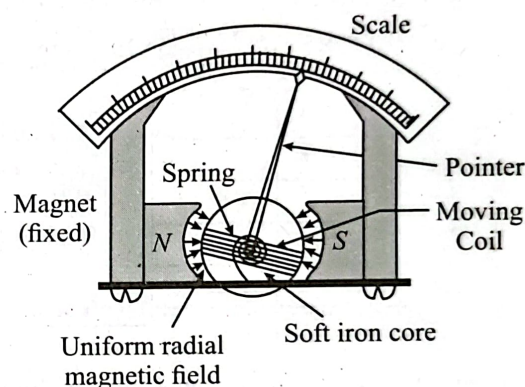
By placing a cylindrical soft iron core inside the coil, the magnetic field is made uniform and radial. Thus, the angle $\theta = 90°$ for any position of the coil. The deflecting torque acting on the coil becomes

$$\tau_d = BINA$$

The coil rotates, the pointer moves on the scale, the spring winds and produces a restoring torque τ_r. If k is the restoring torque per unit twist, and ϕ is the final or steady state deflection (or rotation) of the coil, we have

Deflecting torque τ_d = Restoring torque τ_r

$$BINA = k\phi$$

$$\Rightarrow I = \frac{k}{BIA}\phi$$

or $I = K_G \phi$

Where $K_G = \dfrac{k}{BIA}$ is galvanometer constant.

Thus, we see that the current passing through the coil is directly proportional to its deflection.

By suitable modifications, a galvanometer can be converted into an ammeter or a voltmeter.

Train Your Brain

Example 21: A coil in the shape of an equilateral triangle of side 0.02 m is suspended from a vertex such that it is hanging in a vertical plane between the pole pieces of a permanent magnet producing a horizontal magnetic field of 5×10^{-2} T. Find the couple acting on the coil when a current of 0.1 ampere is passed through it and the magnetic field is parallel to its plane.

Sol. The area of the coil is

$$A = \frac{1}{2} L \times L \sin 60° = \frac{1}{2} \times (0.02)^2 \times \frac{\sqrt{3}}{2} = \sqrt{3} \times 10^{-4} \, m^2$$
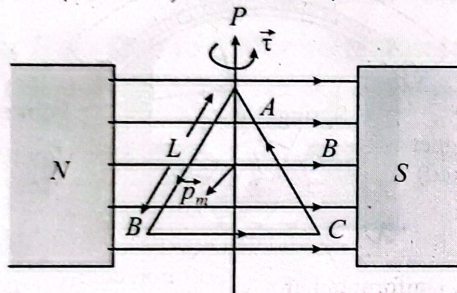
Its magnetic moment,

$$P_m = IA = 0.1 \times \sqrt{3} \times 10^{-4} = \sqrt{3} \times 10^{-5} \, A \, m^2$$

Now the couple on current carrying coil in a magnetic field is given by

$$\vec{\tau} = \vec{p}_m \times \vec{B}$$

or $\tau = p_m B \sin 90° = p_m B$

or $\tau = (\sqrt{3} \times 10^{-5}) \times 5 \times 10^{-2} = 5\sqrt{3} \times 10^{-7} \, N \, m$

Example 22: A electron in an atom is revolving in anticlockwise direction in a circular orbit of radius r. (i) Obtain an expression for the orbital magnetic moment of the electron. (ii) The atom is placed in a uniform magnetic induction $\vec{B}$ such that the plane-normal of the electron orbit makes an angle of 30° with the magnetic induction as shown. Find the torque experienced by the orbiting electron.
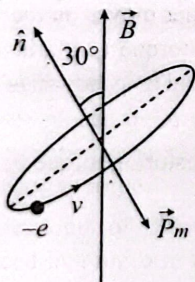
Sol. (i) In case of a charged particle moving in a circle,

$$I = \frac{q}{T} = \frac{-e \, v}{2\pi r}$$

Therefore, the magnetic moment is

$$\vec{P}_m = I\vec{A} = -\frac{e \, v}{2\pi r} \pi r^2 \hat{n} = -\frac{1}{2} evr \, \hat{n}$$

(ii) In case of a current loop in a magnetic field, the torque is given by

$$\vec{\tau} = \vec{p}_m \times \vec{B} = \left(-\frac{evr}{2} \hat{n} \right) \times (\vec{B}) = \frac{evr \, B}{2} \hat{k}$$

where $\hat{k}$ is a unit vector coming of the page.

Concept Application

21. A circular coil of radius 4 cm and 100 turns. In this coil a current of 2A is flowing. It is placed in a magnetic field of 0.1 weber/m^2. The amount of work done in rotating it through 180° from its equilibrium position will be
    (a) 0.1 J   (b) 0.2 J
    (c) 0.4 J   (d) 0.8 J

22. A rectangular loop carrying a current i is situated near a long straight wire such that the wire is parallel to one of the sides of the loop and the plane of the loop is same as that of the left wire. If a steady current I is established in the wire as shown in the figure the loop will
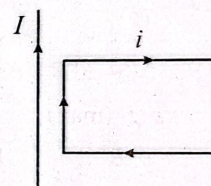
    (a) Rotate about an axis parallel to the wire.
    (b) Move away from the wire.
    (c) Move towards the wire.
    (d) Remain stationary.

23. The magnetic moment of a circular orbit of radius 'r' carrying a charge 'q' and rotating with angular velocity ω is given by
    (a) $\dfrac{q\omega r}{2\pi}$ (b) $\dfrac{q\omega r^2}{2}$
    (c) $\dfrac{q\omega}{r}$ (d) $q\omega r^2$

24. The coil of galvanometer consists of 100 turns and effective area of 1 square cm. The restoring couple is 10^{-8} N-m/rad. The magnetic field between the pole pieces is 5T. The current sensitivity of this galvanometer will be
    (a) 5×10^4 rad/μA
    (b) 5×10^{-6} rad/A
    (c) 2×10^{-7} rad/A
    (d) 5 rad/μA